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(1862 1943)

Using Math: Many students promptly forget their math classes, as soon as they finish them. That's not OK in science. You may have thought during your math class, "Who uses algebra in real life?" Scientists do! In Science, you are literally required to remember and use all of the math that you have ever taken, from kindergarten to calculus. Some science professors are willing to review math, but most are not.

apartment is good for New York, driving 105 kilometers per hour is the highway speed limit, or that a gallon of milk weighs 4 kilograms.

This also means knowledge of the metric prefixes and how they relate - by converting - related units, like meter, kilometer and centimeter.

When scientists measures everything, they often run into extremely large or

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The rounding procedure is to ROUND UP if the next digit is GREATER THAN FIVE (5); ROUND DOWN if the next digit is LESS THAN FIVE (5); and ROUND TO AN EVEN DIGIT if the next digit is EXACTLY FIVE (5 or 500...). Note: this scientific rounding procedure is different than that taught in Math class.

Remember: calculators DO NOT know how to use significant figures.

Remember also the order of operations: if there are no parentheses powers (exponents) are calculated first, multiplication and division next, and addition and subtraction is last.

On the responsibility of computer/calculator use in science and engineering: "If the computer screws up - no, it didn't. You screwed up, because you weren't paying

In Physics, we usually plot position or displacement on a Cartesian coordinate plane using x - y axes. We usually plot velocity on a Cartesian coordinate plane using t - x axes.

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system - all Physics equations assume that you are using the proper unit and they don't work otherwise. If you are given a quantity in a non-standard unit, you must convert.

- Sometimes the algebra turns out to have many, many steps. It is often simpler to plug

In Math, up-is-positive and down-is-negative - always. This may be inconvenient in Physics. If an object is only falling downward, some Physicists will change to down-is-positive, and avoid negative signs. I don't recommend this.

Remember, free-fall motion does not mean only downward motion. Free-fall motion is a vertical motion; whether up or down.

2

Problem 1: The largest twin-engine airplane ever made is the Boeing 777. It can carry 375 passengers over 8400 miles at 550 miles per hour. If a Triple -Seven travels 9000 ft (2750 m) down a runway before taking off at a speed of 165 knots (85 m/s), what is its average acceleration and total takeoff time?

Solution:

given:

unknown:

Problem 2: The Chrysler Pacifica minivan can accelerate from rest through a distance of $\frac{1}{4}$ mile (402 m) in 15.9 s. What is its average acceleration and its speed at the end the quarter-mile?

Solution:

$$x_f = \cancel{x_0} + \cancel{v_0}t + \frac{1}{2}at^2$$

$$a = \sqrt{\frac{2x_f}{t^2}}$$

$$a = \sqrt{\frac{2(402\text{m})}{(15.9\text{s})^2}}$$

$$a = 3.18\text{m/s}^2$$

$$v_f = \cancel{v_0} + at$$

$$v_f = 3.18\text{m/s}^2 (15.9\text{s})$$

$$v_f = 50.6\text{m/s}$$

Problem 3: The Dodge Challenger SRT Hellcat Redeye muscle car can accelerate from rest to 100 mph (44.7 m/s) in 7.4 s. (a) How far does a Challenger travel in this time? A Challenger can brake from 100 mph to a stop in 303 ft (92.4 m). (b) How much time does a Challenger take to stop?

Solution: there are two unrelated parts, with different accelerations

part (a) given: $x_0 = 0$, $v_0 = 0$, $v_f = 44.7\text{m/s}$, $t = 7.4\text{s}$

part (a) unknown: $a = ?$, $x_f = ?$

$$v_f = \cancel{v_0} + at$$

$$a = \frac{v_f}{t}$$

$$a = \frac{44.7\text{m/s}}{7.4\text{s}}$$

$$a = 6.04\text{m/s}^2$$

$$x_f = \cancel{x_0} + \cancel{v_0}t + \frac{1}{2}at^2$$

part (b) given:

part (b) unknown:

Problem 4: Although the US Navy's 100,000 ton

Problem 5: A driver traveling at 10 m/s spots a red light 40 m ahead, and steps on the brake after a one second reaction time. What deceleration would stop the car right at the light?

Solution: this is a two part related motion: reaction time has zero acceleration. The reaction time distance is the initial position for the braking.

Part 1: given: $a_1 = 0$, $v_0 = 10\text{ m/s}$, $v_1 = 10\text{ m/s}$, $x_0 = 0$, $t_0 = 0$, $t_1 = 1\text{ s}$
 unknown: $x_1 = ?$

$$\begin{aligned}x_1 &= v_1 t_1 \\x_1 &= 10\text{ m/s}(1\text{ s}) \\x_1 &= 10\text{ m}\end{aligned}$$

Part 2: given: $v_1 = 10\text{ m/s}$, $v_f = v_2 = 0$, $x_1 = 10\text{ m}$, $x_f = x_2 = 30\text{ m}$, $t_1 = 1\text{ s}$
 unknown: $a_2 = ?$

$$\begin{aligned}v_2^2 &= v_1^2 + 2a_2(x_2 - x_1) \\a_2 &= \frac{-(v_1^2)}{2(x_2 - x_1)}\end{aligned}$$

Problem 6: A speeder zooms past an idling police car at 20 m/s. If the police officer instantly starts chasing the speeder with a constant acceleration of 4 m/s², how long will it be before the police car overtakes the speeder? How far does the speeder travel before being overtaken?

Solution: there are two separate but related motions. You should set up the speeder and police car separately and relate them together because the police car must have the same total time and distance as the speeder.

given: $v_s = 20 \text{ m/s}$, $a_s = 0$, $v_{0,p} = 0$, $a_p = 4 \text{ m/s}^2$, $t_s = t_p$, $x_{0,s} = x_{0,p} = 0$, $x_{f,s} = x_{f,p}$

unknown: $t_s = t_p = ?$, $x_s = x_p = ?$

speeder (no acceleration): $x_{f,s} = v_s t_s$

police car: $x_{f,p} = x_{0,p} + v_{0,p} t_p + \frac{1}{2} a_p t_p^2$

$$x_{f,s} = x_{f,p}$$

$$v_s t_s = \cancel{x_{0,p}} + \cancel{v_{0,p} t_p} +$$

Problem 7: A speeder zooms past a parked police car at 20 m/s. If the police officer starts chasing the speeder with an acceleration of 4 m/s^2 after a 2 second reaction time, how long will it be before the police car overtakes the speeder?

Solution: this is also are two separate but related motions. The police car reaction time makes the speeder and police car driving times different. You should set up the speeder and police car separately and relate them together because the police car must still have the same total distance as the speeder.

given: because of reaction time

and

unknown:

Problem 8: At the climax of the 1933 movie, and the 2005 remake, King Kong fell from

Problem 9: Standing on the middle of the George Washington Bridge, you drop a rock. Since the rock splashes into the Hudson River after 3.72 s, how high is the bridge? How fast is the rock moving as it hits the water? Ignore air resistance. (212+10ft≈67.7m)

Solution: the origin can be chosen as either the bridge level or the river level. Here, bridge level is chosen.

given: $g = 9.8 \text{ m/s}^2$, $y_0 = 0$, $v_0 = 0$, $t = 3.72 \text{ s}$

unknown: $h_{\text{bridge}} = |y_{\text{rock}}| = ?$, $v_f = ?$

$$y_{\text{rock}} = \cancel{y_0} + \cancel{v_0}t - \frac{1}{2}gt_{\text{rock}}^2$$

$$y_{\text{rock}} = -\frac{1}{2}(9.8 \text{ m/s}^2)(3.72 \text{ s})^2$$

$$y_{\text{rock}} = -67.8 \text{ m}$$

$$h_{\text{bridge}} = 67.8 \text{ m}$$

$$v_f = \cancel{v_0} - gt$$

$$v_f = -9.8 \text{ m/s}^2(3.72 \text{ s})$$

$$v_f = -36.5 \text{ m/s}$$

Problem 10: Standing on the middle of the Verrazano Bridge on a foggy day, you cannot

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Problem 11: A batter hits a baseball from 1 m above the ground at 35m/s straight upward. It is a high pop up. How high does the ball rise above the ground? If the ball is allowed to drop to the ground, how long would it take and how fast is it moving when it hits?

Solution: ground level is chosen as the origin
given:

unknown:

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Problem 12: You toss a ball straight upward at 11 m/s to a friend, who catches it on a balcony 6 m above you. How fast is the ball moving when caught, and how long will take to be caught?

Solution: given:

unknown:

Note: there are two answers, depending on whether the ball is caught on the way up (the positive velocity) or is falling back down (the negative velocity) when caught. The

$$\vec{A} \times \vec{B} \quad \vec{B} \times \vec{A}$$

the cross product is anti-commutative

→

Since matrix multiplication is tedious, there is a shortcut for the 3-dimensional (ONLY) cross product:

vector cross product magnitude-only shortcut:

vector cross product direction-only shortcut (the right-hand-rule, RHR):

Hold your right hand with fingers straight and thumb out;

step 1: point your fingers in the direction of vector A,

step 2: point your thumb in the direction of vector B,

step 3: your palm faces the the direction of vector R.

Note: there are alternate ways to set up the RHR.

Note: the direction of the vector cross product resultant R is always perpendicular to the plane formed by the original two vectors A and B. If A and B are on the x-y plane, R is along the z-axis. The three vectors exist in a three-dimensional space.

Two-dimensional motion is motion that is not strictly on a straight line. Rather it combines up/down motion with left/right motion.

Projectile motion is most typical of two dimensional motion: an object is thrown into the air, but not straight up or down, so that as gravity pulls the object downward, it also moves horizontally.

Therefore, the initial velocity vector is actually a total of the initial horizontal and initial vertical velocities. Computing with the initial velocity vector is not really feasible.

Instead, the initial velocity must be broken up as a vector quantity into its horizontal and vertical components: v_{0x} and v_{0y} . These individual components are used to solve

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Problem 2: Before the invention of gunpowder cannon, the most powerful weapon in the world was the trebuchet, a sling, lever and counterweight machine that could smash castle walls by hurling quarter-ton boulders the length of three football fields. If a

Problem 5: What must be the initial speed of a basketball jump shot, if it is thrown from a height of 2.1 m, at an angle of 50° so that the ball swishes into the 3.0 m high basket, 5.2 m away?

Solution:

given: $\theta_0 = 50^\circ$, $x_0 = 0\text{ m}$, $y_0 = 2.1\text{ m}$, $x_f = 5.2\text{ m}$, $y_f = 3.0\text{ m}$, $v_{fy} = -v_{0y}$

unknown: $v_0 = ?$

$$y_f = y_0 + v_{0y}t - \frac{1}{2}gt^2$$

$$y_f = y_0 + v_0 \sin \theta_0 t - \frac{1}{2}gt^2$$

$$y_f = y_0 + \cancel{v_0} \sin \theta_0 \frac{x}{\cancel{v_0} \cos \theta_0}$$

$$x = v_{0x}t$$

$$\frac{x}{v_{0x}} = t$$

$$t = \frac{x}{v_0 \cos \theta_0}$$

Notice that this question is asking for INITIAL speed. The question is sort of "backwards." This tends to complicate the Math and I did not complete the algebra, before plugging in the data. I simplified by doing some arithmetic first. "Arithmetic is easier than algebra."

Problem 6: The US Navy's 9700 ton World War II-era cruiser, USS Brooklyn (built at the New York [Brooklyn] Navy Yard), was armed with fifteen 6 inch/47 caliber cannon that fired 130 lb shells at a velocity of 2500 ft/s (762 m/s) to a range of 26,100 yards (14.8 miles; 23,900 m). What angle of elevation did Brooklyn fire her guns to reach this range? Ignore air resistance. Hint: $2 \sin \theta \cos \theta = \sin 2\theta$. (PHY 1300 only)

Solution: given: $|v_0| = 762 \text{ m/s}$, $x_f = 23,900 \text{ m}$, $y_0 = y_f = 0$, $v_{fy} = -v_{0y} = -v_0 \sin \theta_0$
 unknown: $\theta_0 = ?$

$$x = v_{0x}t$$

$$x = v_0 \cos \theta_0 t$$

$$t = \frac{x}{v_0 \cos \theta_0}$$

$$\text{and } v_{fy} = v_{0y} - gt$$

$$-v_{0y} = v_{0y} - gt$$

$$2v_0 \sin \theta_0 = gt$$

$$t = \frac{2v_0 \sin \theta_0}{g}$$

$$\theta_0 = \frac{1}{2} \sin^{-1} \left(\frac{xg}{v_0^2} \right)$$

$$\theta_0 = \frac{1}{2} \sin^{-1} \left(\frac{23,900 \text{ m} (9.8 \text{ m/s}^2)}{(762 \text{ m/s})^2} \right)$$

$$\theta_0 = 11.9^\circ$$

$$\text{combined } \frac{x}{v_0 \cos \theta_0} = \frac{2v_0 \sin \theta_0}{g}$$

$$x = \frac{2v_0^2 \sin \theta_0 \cos \theta_0}{g}$$

$$x = \frac{v_0^2 \sin 2\theta_0}{g}$$

Note: Brooklyn actually fired at 47.5° for maximum range, because of air resistance.

Problem 7: An airliner traveling from Chicago to New York with an airspeed of 250 m/s needs to fly due east, but encounters a steady 20 m/s crosswind blowing at 50° north of west. What heading should the airliner take and what would be its groundspeed?

Solution: given and unknown:

$$|v_{air}| = 250 \text{ m/s}, \theta_{air} = ?$$

$$|v_{wind}| = 20 \text{ m/s}, \theta_{wind} = 130^\circ$$

Beginning with horizontal components:

The v_{ground} and v_{air} are both unknown. Check vertical components to see if the situation simplifies.

Now, bring v_{air} back to horizontal components.

$$\left| v_{\text{ground}} \right| \cos 0 = (250 \text{ m/s}) \cos (-3.50) + (20 \text{ m/s}) \cos 130^\circ$$

$$\left| v_{\text{ground}} \right| = 249.5 \text{ m} + (-12.9 \text{ m/s})$$

$$\left| v_{\text{ground}} \right| = 236.6 \text{ m/s}$$

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(1642 1727)



Mechanics asks why do objects move?

Newton's Laws of Motion

The First Law - the law of equilibrium - an object at rest will stay at rest, an object in motion will stay in motion - no net force = no acceleration (change of motion).

The Second Law - the law of inertia; "force causes acceleration" - yes net force = yes acceleration.

the sum of force vectors (acting on a mass) equals the mass times its acceleration

This is the important law, because we be using it a lot for problem solving.

The Third Law - the law of action-reaction - for every action, there is an equal but opposite reaction - if any force is applied to an object, that object tends to resist with a opposite force.

Be careful: weight and mass are not the same thing in Physics: weight is the force of gravity; mass measures "inertia" - resistance to change of motion.

sum. The forces must be identified before they can be added - as vectors. This is the important part of the Physics, the rest is "just Math".

Forces are identified on a free-body diagram. A free-body diagram reduces a mass to a point at the origin of an x-y co-ordinate plane, and draws all the forces (separated into vector components as necessary) acting on the object as arrows pulling out from that point. Make the diagram neat, to reasonable scale and fairly large, so that you can write on and read off your information. Often, the forces can be determined and we need to find the acceleration. With the acceleration in hand, we can use it in a motion problem.

There are four major forces that you should check for:

- 1) EXTERNAL FORCES are any forces that are not a property of the object itself. They are applied to the object from outside of the object.
- 2) WEIGHT is the force of gravity.

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individually. In real life engineering, this is done mostly on computers. Computer aided design/engineering/manufacturing software is standard in modern engineering.

Example: A 2000 kg elevator is pulled upward by a cable. If the tension force is 22,000 N, what is the elevator's acceleration? Ignore air resistance?

Solution:

given:

unknown:

Free-body diagram

Newton's Second Law

negative, is because Rene Descartes said so 400 years ago. If that's not convenient, don't do it that way." Let positive follow the direction of motion.

4

Problem 1: If a 50 kg mass experiences a net force of $\vec{F} = (150N)i + (100N)j$, what is the acceleration of the mass? (PHY 1300 only.)

Solution:

given: $m = 50kg$, $F = (150N)i + (100N)j$

unknown: $a = ?$

$$\sum \vec{F} = m$$

Problem 3: A 100 kg wooden crate is at rest on a horizontal floor with a coefficient of static friction of 0.70 and coefficient of kinetic friction of 0.50. (a) What is the minimum horizontal

part (b) given: $m = 100\text{kg}$, $\mu_k = 0.50$, $\theta = 0^\circ$, $F = 686\text{N}$,

free-body diagram looks the same as part a

part (b) unknown: $a = ?$

$$\begin{aligned} F &= 686\text{N} \\ mg &= 980\text{N}, \\ F_N &= mg = 980\text{N}, \\ F_{fr} &= \mu_k F_N = 490\text{N} \end{aligned}$$

$$\begin{aligned} \sum F_x &= ma_x \\ F - F_{fr} &= ma_x \\ a_x &= \frac{F - \mu_k mg}{m} \\ a_x &= \frac{F - \mu_k mg}{m} \\ a_x &= \frac{686\text{N} - 490\text{N}}{100\text{kg}} \\ a_x &= 1.96\text{m/s}^2 \end{aligned}$$

Problem 4: A horizontal force presses a 4 kg textbook against a

Problem 7: If you dragged a 50 kg box up a 30° inclined plane, with a 400 N force parallel to the incline, how quickly would the box accelerate? Assume that there is 0.35 coefficient of friction.

Solution:

given:

$$m = 50\text{kg}, \theta = 30^\circ, F = 400\text{N}, \mu = 0.35$$

unknown: $a = ?$

Free-body diagram: Since the normal force is perpendicular to the incline, the normal force is not vertical, and is not opposite to the weight.

$$\begin{aligned} F &= 400\text{N} \\ mg &= 490\text{N}, \\ mg \sin \theta &= 245\text{N}, \\ mg \cos \theta &= 424\text{N}, \\ F_N &= mg \cos \theta = 424\text{N}, \\ F_{fr} &= \mu F_N = 148\text{N} \end{aligned}$$

Newton's Second Law:

$$\begin{aligned} \sum F_x &= ma_x \\ F - mg \sin \theta - F_{fr} &= ma_x \\ a &= \frac{F - mg \sin \theta - F_{fr}}{m} \end{aligned}$$

Problem 8: An Atwood machine consists of two masses connected by a cord, that is hung over a perfect pulley. If

$$\begin{aligned}
 \cancel{F_T} - m_A g &= m_A a \\
 + (m_B g - \cancel{F_T} &= m_B a) \\
 \hline
 m_B g - m_A g &= m_A a + m_B a
 \end{aligned}$$

$$a = \frac{g(m_B - m_A)}{m_A + m_B}$$

$$a = \frac{9.8 \text{ m/s}^2 (15 \text{ kg} - 10 \text{ kg})}{10 \text{ kg} + 15 \text{ kg}}$$

$$a = 1.96 \text{ m/s}^2$$

using mass A to find F_T :

$$\begin{aligned}
 F_T - m_A g &= m_A a \\
 F_T &= m_A a + m_A g \\
 F_T &= m_A (a + g) \\
 F_T &= 10 \text{ kg} (9.8 \text{ m/s}^2 + 1.96 \text{ m/s}^2) \\
 F_T &= 118 \text{ N}
 \end{aligned}$$

using mass B to find F_T :

$$\begin{aligned}
 m_B g - F_T &= m_B a \\
 F_T &= m_B g - m_B a \\
 F_T &= m_B (g - a) \\
 F_T &= 15 \text{ kg} (9.8 \text{ m/s}^2 - 1.96 \text{ m/s}^2) \\
 F_T &= 118 \text{ N}
 \end{aligned}$$

Problem 9: Two masses are connected by a cord. One mass is on a table. The other mass is hung over a perfect pulley. If mass A is 15 kg and mass B is 12 kg, what is the minimum coefficient of static friction of the table that will keep system from moving when mass B is released?

Solution:

given: $m_A = 12\text{kg}$, $m_B = 15\text{kg}$, $a = 0$

unknown: $\mu_s = ?$

Two masses needs two free-body diagrams. Assume mass B has tendency to fall for positive/negative directions.

Two free-body diagrams means two Newton's Second Law calculations that can be then combined, because their accelerations are the same.

$$\begin{aligned} \sum F_x &= m_A a_x & \sum F_y &= m_B a_y \\ F_T - F_{fr} &= m_A a & m_B g - F_T &= m_B a \\ F_T - \mu_s m_A g &= 0 & m_B g - F_T &= 0 \end{aligned}$$

5:

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Circular (or centripetal) motion means turning around in a circle.

Turning is not the same as spinning "It's perfectly OK for you to turn your car around in a circle, but if your car starts spinning, you're about to crash."

Since velocity is a vector - a two part measurement - when we say that acceleration is the change of velocity, we can change either part.

Centripetal (or radial) acceleration is change of velocity DIRECTION; direction change is inward on radius line. Its magnitude is:

$$|a_c| = \frac{|v|^2}{r}$$

Linear (or tangential) acceleration is change of velocity MAGNITUDE; change is along a straight tangent line.

"We will not be changing both velocity magnitude and direction at the same time in this class. (Under normal circumstances.) Remember, you are taught in driver's ed NOT to step on the gas or brake AND turn the steering wheel at the same time."

Since we have a special version of acceleration, there is a centripetal force version of Newton's Second Law

$$\sum F_c = m \frac{v^2}{r}$$

Problem 2: The Mazda Miata sports car is not very swift, but it is very nimble. If there is a coefficient of friction of 0.95 between a Miata's tires and asphalt, how fast can a Miata safely drive around a 100 m radius curve?

Solution:

given:

unknown:

Free-body diagram: Remember, "centripetal" means "toward the center." Centeryhe

Problem 3: A regular pendulum swings back-and-forth in an arc. A conical pendulum swings around in a horizontal circle. If a conical pendulum swings around a

Newton's Second Law: horizontal circle

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Remember, the r is the radial distance (direct line) measured from the center of a planet, not from the surface (the height).



Projectile motion, as discussed earlier, assumes the world is flat. The Earth is not flat. If a projectile is thrown less than a few thousand meters, we can pretend the Earth is flat. If a projectile is thrown thousands of kilometers, we must account for the spherical Earth curving down under the projectile. If a projectile is thrown correctly, it will continually “miss” hitting the Earth and end up circling the Earth. Satellite motion is a

Problem 2: As the Earth orbits the Sun, and the Moon orbits the Earth, they will align at a right angle every two weeks. What is the net gravitational force (magnitude and direction) on the Earth, from the Sun and Moon, in this alignment?

The masses of the Earth, Moon and Sun are $5.98 \times 10^{24} \text{ kg}$, $7.35 \times 10^{22} \text{ kg}$, and $1.99 \times 10^{30} \text{ kg}$, respectively. The Earth-Sun and Earth-Moon distances are $1.50 \times 10^{11} \text{ m}$, and $3.84 \times 10^8 \text{ m}$. The Gravitational Constant is $6.67 \times 10^{-11} \text{ N m}^2/\text{kg}^2$

Solution:

given:

$$m_{\text{Earth}} = 5.98 \times 10^{24} \text{ kg}, m_{\text{Moon}} = 7.35 \times 10^{22} \text{ kg}, m_{\text{Sun}} = 1.99 \times 10^{30} \text{ kg},$$
$$r_{\text{Earth-Sun}} = 1.50 \times 10^{11} \text{ m}, r_{\text{Earth-Moon}} = 3.84 \times 10^8 \text{ m}$$

unknown: $F_{E \text{ net}} = ?$

Free-body diagram for gravitational forces:

Universal Gravitation:

$$F_{ES} = G \frac{M_E M_S}{r_{ES}^2}$$

$$F_{ES} = 6.67 \times 10^{-11} N \cdot m$$

Problem 3: Popular accounts of the 1968 - 1972 Apollo voyages to the Moon often say that the spacecraft were rocketed all the way to the Moon. This is wrong. The Apollo spacecraft needed only to reach the gravitational force "equilibrium point" between the Earth and the Moon, after which the spacecraft would simply "fall" the rest of the way to the Moon. What is the equilibrium distance along a straight line between the Earth and the Moon? The masses of the Earth and Moon are $5.98 \times 10^{24} \text{ kg}$ and $7.35 \times 10^{22} \text{ kg}$, respectively. The Earth-Moon distance is $3.84 \times 10^8 \text{ m}$. The Gravitational Constant is $6.67 \times 10^{-11} \text{ N m}^2/\text{kg}^2$

Solution:

given: $m_{\text{Earth}} = 5.98 \times 10^{24} \text{ kg}$, $m_{\text{Moon}} = 7.35 \times 10^{22} \text{ kg}$,
 $r_{\text{Earth-eq}} = x$, $r_{\text{Moon-eq}} = 3.84 \times 10^8 \text{ m} - x$

$$\begin{aligned} F_{\text{Apollo-Earth}} &= F_{\text{Apollo-Moon}} \\ G \frac{m_{\text{Apollo}} m_{\text{Earth}}}{r_{\text{Earth-eq}}^2} &= G \frac{m_{\text{Apollo}} m_{\text{Moon}}}{r_{\text{Moon-eq}}^2} \\ \frac{m_{\text{Earth}}}{r_{\text{Earth-eq}}^2} &= \frac{m_{\text{Moon}}}{r_{\text{Moon-eq}}^2} \end{aligned}$$

$$\frac{5.98 \times 10^{24}}{x^2} = \frac{7.35 \times 10^{22}}{(3.84 \times 10^8 - x)^2}$$

$$\frac{2.4454 \times 10^{12}}{x} = \frac{2.7111 \times 10^{11}}{3.84 \times 10^8 - x}$$

$$9.3903 \times 10^{20} - 2.4454 \times 10^{12} x = 2.7111 \times 10^{11} x$$

$$9.3903 \times 10^{20} = 2.7165 \times 10^{12} x$$

$$3.457 \times 10^8 \text{ m} = x = r_{\text{Earth-eq}}$$

$$\text{or } r_{\text{Moon-eq}} = 3.84 \times 10^8 \text{ m} - 3.457 \times 10^8 \text{ m} = 3.83 \times 10^7 \text{ m}$$

7: ,

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The work-energy theorem is a mathematical statement of the work-energy relationship. There are different ways to write the theorem.

It is called a theorem, because it can be derived from Newton's Second Law. You are not responsible for the mathematical proof.

We need you to recognize that Newton's Second Law force and acceleration calculations are vector calculations; while work-energy theorem calculations are scalar calculations. They are Physically equivalent - they give the same answers. Many work-energy problems will look familiar, because they are. We want you to see they give the same answers. In real life, the technique used depends on which is easier. Usually, but not always, the work-energy theorem will be easier, because scalar math is simpler.



In science, there are fundamental concepts called the conservation laws. In science, "conservation" means "does not change." We having been studying "change" in earlier topics: velocity is change of position, acceleration is change of velocity; force causes acceleration, so force causes change. etc.

However, if everything is always changing, we don't know anything for sure - what is true today, changes overnight and is longer true tomorrow. Therefore, science also searches for the things do not change; that are certain. These are the conservation laws. I always say that the conservation laws are "the things that are eternal."

Be careful, conservation of energy only applies in specific situations - work adds energy into a system, friction "steals" energy and dissipates it as heat. We say external and friction forces are nonconservative forces. Since there is always some friction in real life, conservation of energy is an approximation in real life.

In this class, roller coasters, pendulums, springs and projectile motion normally obey conservation of energy.

Power is rate of work produced or energy used. It is mathematically defined as:

$$P = \frac{W}{t}$$

"Power" is also an ordinary English word, used in regular conversation, that also has absolutely nothing to do with what they mean in Physics. Do not mix up English and Physics.

7 ,

Problem 1: A tow-truck's cable pulls a 1800 kg car at constant speed, a distance of 12 km on a level road, against a 0.15 coefficient of friction. How much work is done by the

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Problem 4: A tow-truck's cable pulls a 1800 kg car with a 5000 N tension force, 150 m up along a 10° ramp, against a 2000 N friction force. If the tow begins at 10 m/s at the bottom of the ramp, how fast will the car be moving when it reaches the top of the ramp? Assume the cable is parallel to the incline.

Solution:

given:

from free-body diagram and Newton's Second Law:

from the right triangle:

unknown:

Work-energy theorem:

Problem 5: The first hill of the Cyclone roller coaster at Coney Island climbs up 85 ft (26 m) before dropping to ground level, and then rises again to a second 70 ft (21 m) hill. If a roller coaster car is released at the top of the first hill at 5 m/s, what would be the speed of the car when it reaches the bottom of the hill? What would be the speed of the car when it reaches the top of the second hill? Ignore friction.

Solution:

given: $v_{hill1} = 5\text{ m/s}$, $h_{hill1} = 26\text{ m}$, $h_{hill2} = 21\text{ m}$

unknown: $v_{valley} = ?$, $v_{hill2} = ?$

$hill_1 = valley$

$$KE_i + PE_i = KE_f + \cancel{PE_f}$$

$$\frac{1}{2}mv_i^2 + mgh_i = \frac{1}{2}mv_f^2$$

$$v_f = \sqrt{v_i^2 + 2gh_i}$$

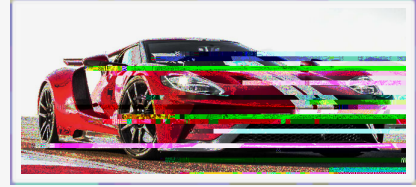
$$v_f = \sqrt{(5\text{ m/s})^2 + 2(9.8\text{ m/s}^2)(26\text{ m})}$$

$$v_f = 23.1\text{ m/s}$$

$$hill_1 = hill_2$$

Problem 7: Before the invention of gunpowder cannon, the most powerful weapon in the world was the trebuchet, a sling, lever and counterweight machine that could smash

Problem 9: The 3380 lb (1535 kg) Ford GT supercar can accelerate from rest to 150 mph (67.1 m/s) in 14.5 s. How powerful is the GT's engine?



Solution:

given: $m = 1535\text{kg}$, $v = 67.1\text{m/s}$, $t = 14.5\text{s}$

unknown: $P = ?$

$$P = \frac{W}{t}$$

$$P = \frac{KE + \cancel{PE}}{t}$$

$$P = \frac{\frac{1}{2}mv^2}{t}$$

$$P = \frac{\frac{1}{2}(1535\text{kg})(67.1\text{m/s})^2}{14.5\text{s}}$$

$$P = 2.38 \times 10^5 \text{kg m}^2/\text{s}^3$$

$$P = 2.38 \times 10^5 \text{Watt} \quad 320 \text{ horsepower}$$

Note: since the GT's engine is rated at 647 hp, the losses to air resistance, tire slippage, etc., are over 50%.

Problem 11: During World War 2, Grumman Aircraft of

8:

. \hat{A} (1920 1992)

Momentum is another word that has everyday English meanings that have nothing to do with Physics. In fact, there is no real way to describe, in words, what momentum means in Physics. The definition is purely mathematical. The definition of linear momentum is:

Notice, since velocity is a vector, momentum is also a vector. In other words, pay attention to the direction and positive and negative signs. Also pay attention to the difference between horizontal and negative.

Newton's Second Law was originally written by Newton in terms of momentum:

8

Problem 1: The most powerful aircraft cannon ever flown is carried by the US Air Force's A-10, called the Warthog. The Warthog uses its 30mm Avenger cannon to kill tanks. The Avenger cannon is reputed to be so powerful that firing it produces enough recoil to noticeably slow a 'Hog in flight. Since the Avenger fires 13.9 oz (395 g) bullets at a rate of 3900 rounds per minute (in 2-3 second bursts) and a velocity of 3500 ft/s (1067 m/s), is this true? Assume that "noticeable" means an average recoil force greater than the thrust the Warthog's two TF-34 engines: 18,130 lbs force \approx 80,600 N.

Solution:

given:

unknown: $F = ?$

No; the 27,400 N recoil force is less than the 80,900 thrust force.

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Problem 2: If a 100 kg linebacker, running at 7.5 m/s, tackles a 150 kg tackling sled, initially at rest, how fast would the combination be immediately moving after the collision?

9:

□

,

,

$$\begin{aligned} \theta_f &= \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2 \\ \omega_f &= \omega_0 + \alpha t \\ \alpha_f &= \alpha_0 \end{aligned}$$

Compare these definitions and formulae to those introduced before. Can you see that they are not new? Pay attention to positive versus negative rotation direction. We will continue to follow the conventional angle direction - counterclockwise rotation is positive and clockwise rotation is negative.

ROLLING combines linear and rotational motion. A wheel does not spin in place - as a wheel spins, its axis moves linearly. The radius connects linear and rotational motion.

$$\begin{aligned} \theta &= \frac{s}{r} \\ &= \frac{v}{r} \\ &= \frac{a}{r} \end{aligned}$$

The shape of an object affects the way it will spin. The moment of inertia replaces mass, but depends on the mass and shape.

$$I = \int r^2 dm \quad (\text{PHY 1300 only})$$

$$\text{OR } I = kmr^2 \text{ where } k \text{ is the shape constant}$$

for some simple shapes:

$$\begin{aligned} I_{\text{disk}} &= \frac{1}{2} mr^2 \\ I_{\text{solid sphere}} &= \frac{2}{5} mr^2 \\ I_{\text{hollow sphere}} &= mr^2 \\ I_{\text{hoop}} &= mr^2 \end{aligned}$$

For example: a solid wheel is a disk, a baseball is a solid sphere, a basketball is a hollow sphere, a bicycle wheel is a hoop.

center of mass of simple multi-mass objects (PHY 1300 only)

$$x_{cm} = \frac{\sum x_i m_i}{M}$$

We continue to replace linear measurements with angular measurements. We substitute TORQUE (τ -)for force:

$$\tau = r F = rF \sin \phi_{rF}$$

Strictly speaking, torque is computed as a cross product (PHY 1300 only)

$$\vec{\tau} = \vec{r} \times \vec{F}$$

This means torque exists in a 3-dimensional space. It is normally not necessary to worry about (yet). The magnitude is normally enough.

$$|\vec{\tau}| = |\vec{r} \times \vec{F}| = |r| |F| \sin \phi_{rF}$$

Just pay attention to a positive counterclockwise versus a negative clockwise rotation direction.

Since torque replaces force, we have a new version of Newton's Second Law

$$\sum \vec{\tau} = I \vec{\alpha}$$

Since rotation is a different type of motion, we also have rotational kinetic energy

$$KE = \frac{1}{2} I \omega^2$$

The work-energy theorem and conservation of energy is still valid.

Angular momentum is:

$$L = I \omega$$

Conservation of momentum is still valid.

Since "Everything is the same as before; it just looks different" recognize that rotation problems are solved in the exact same way as before. Learn to be comfortable with the new notations.

9

Problem 1: A Ford Mustang GT pony car can accelerate from rest to 100 mph (44.7 m/s) in 9.5 s. (a) What is the angular acceleration of its 70 cm diameter wheels? (b) How many revolutions do the wheels make in this time? (c) How fast are the wheels spinning at the end of the 9.5 s?

Solution:

given: $v_0 = 0$, $\theta_0 = 0$, $v_f = 44.7 \text{ m/s}$, $t = 9.5 \text{ s}$, $r = \frac{d}{2} = 0.35 \text{ m}$

unknown: $a = ?$, θ_f in rev = ?, $f = ?$

part (a):

$$v_f = v_0 + at$$

$$a = \frac{v_f}{t} = \frac{44.7 \text{ m/s}}{9.5 \text{ s}} = 4.71 \text{ m/s}^2$$

$$a = \frac{a}{r} = \frac{4.71 \text{ m/s}^2}{0.35 \text{ m}} = 13.4 \text{ rad/s}^2$$

part (b):

$$\theta_f = \theta_0 + v_0 t + \frac{1}{2} a t^2$$

$$\theta_f = \frac{1}{2} (13.4 \text{ rad/s}^2) (9.5 \text{ s})^2 = 605 \text{ rad} \times \frac{1 \text{ rev}}{2\pi \text{ rad}} = 96 \text{ rev}$$

$$\theta_f = 605 \text{ rad}$$

part (c):

$$f = \frac{v_f}{r} = \frac{44.7 \text{ m/s}}{0.35 \text{ m}} = 127 \text{ rad/s}$$

OR

$$f = \omega + \alpha t = 13.4 \text{ rad/s}^2 (9.5 \text{ s}) = 127 \text{ rad/s}$$

Problem 2: A 2 kg block is attached below a 6 kg, 50 cm diameter solid pulley. When the block is released from rest, it unwinds string off the pulley. What length of string will be pulled off the pulley after the block drops for 2 s?

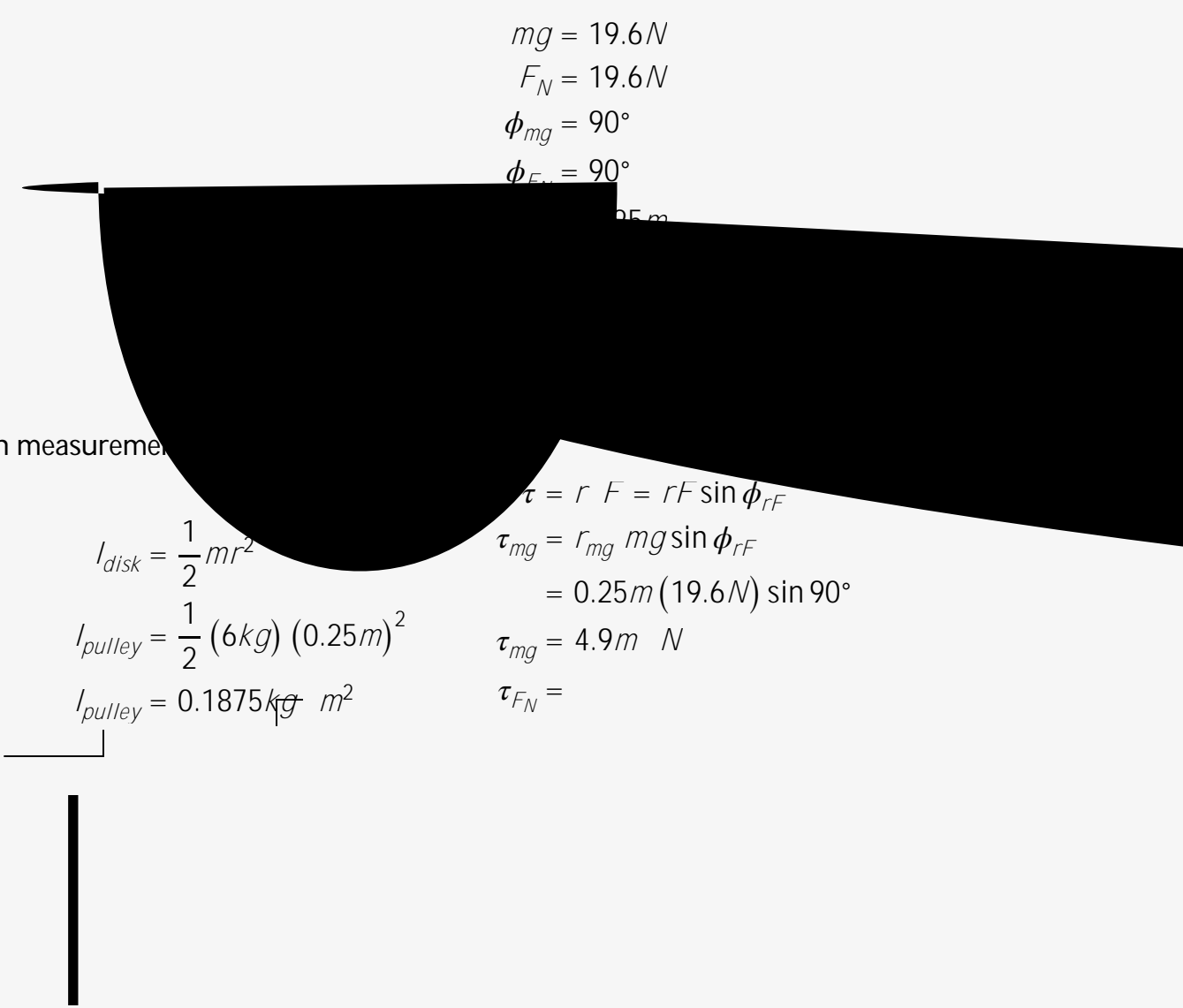
Solution:

given:

$$m_b = 2\text{ kg}, m_p = 6\text{ kg}, r = \frac{d}{2} = 0.25\text{ m}, v_0 = 0, \omega_0 = 0, t = 2\text{ s}$$

unknown: $s_f = ?$

Free-body diagram:



$$mg = 19.6\text{ N}$$

$$F_N = 19.6\text{ N}$$

$$\phi_{mg} = 90^\circ$$

$$\phi_{F_N} = 90^\circ$$

rotation measurement

$$I_{disk} = \frac{1}{2} mr^2$$

$$I_{pulley} = \frac{1}{2} (6\text{ kg}) (0.25\text{ m})^2$$

$$I_{pulley} = 0.1875\text{ kg m}^2$$

$$\tau = r F = rF \sin \phi_{rF}$$

$$\begin{aligned} \tau_{mg} &= r_{mg} mg \sin \phi_{rF} \\ &= 0.25\text{ m} (19.6\text{ N}) \sin 90^\circ \end{aligned}$$

$$\tau_{mg} = 4.9\text{ m N}$$

$$\tau_{F_N} =$$

Newton's Second Law

$$\begin{aligned}\sum \tau &= I \\ \tau_{FN} &= I \\ &= \frac{\tau_{FN}}{I} \\ &= \frac{4.9m \text{ N}}{0.1875k}\end{aligned}$$

motion formula:

Problem 3: A 4.5 kg, 30 cm diameter bowling ball rolls, without slipping, down a 5 m high inclined plane. If the ball rolls from rest at the top, what will be its linear speed at the bottom of the incline?

Solution: if you've never bowled before, a bowling ball is a solid sphere.

given:

unknown:

There is more than one way to solve this problem. Mathematically, the simplest is considered the use of conservation of energy.

since

and

10:

. (1903 1957)

Static equilibrium says "Nothing moves because everything balances."

10

Problem 1: Two 10 kg masses are placed on a 20 kg, 5 m uniform horizontal beam. The beam has two supports. See the figure. What are the two support forces?

Solution:

given: $m_b = 20\text{kg}$, $m_A = 10\text{kg}$, $m_B = 10\text{kg}$

unknown: $F_1 = ?$, $F_2 = ?$

Free-body diagram (find the forces): since there is a possibility of rotation, it is inappropriate to use an x-y plane. You should use a line to represent the beam and place the forces along the line where they belong. Theoretically, you can put the pivot point anywhere you want. To keep things simple, I always choose the pivot at the first support from the left, and measure all radii from there. The center of gravity is the geometric center of a uniform or homogenous object. Objects are uniform in this class, unless specifically stated otherwise.

The diagram's geometry gives:

Find the torques: since all angles are 90° , all $\sin = 1$

$$\tau = rF\sin\phi$$

$$\tau_{F_1} = \cancel{r} F_1 = 0$$

$$\tau_{m_Ag} = r_A m_Ag = 1m(10kg)(9.8m/s^2) = 98m \text{ N (CW)}$$

$$\tau_{m_bg} = r_b m_bg = 1.5m(20kg)(9.8m/s^2) = 294m \text{ N (CW)}$$

$$\tau_{F_2} = r_2 F_2 = 2m(F_2) = 2F_2 m \text{ N (CCW)}$$

$$\tau_{m_Bg} = r_B m_Bg = 3m(10kg)(9.8m/s^2) = 294m \text{ N (CW)}$$

Now that the Physics is done; we're ready for the Math:

First, balance the torques:

$$\sum \tau = 0$$

$$\tau_{F_2} - \tau_{m_A} - \tau_{m_b} - \tau_{m_B} = 0$$

$$\tau_{F_2} = \tau_{m_A} + \tau_{m_b} + \tau_{m_B}$$

$$2F_2 = 98 + 294 + 294$$

$$2F_2 = 686$$

$$F_2 = 343m \text{ N}$$

Second, balance the vertical forces:

$$\sum F_y = 0$$

$$F_1 + F_2 - m_Ag - m_bg - m_Bg = 0$$

$$F_1 = m_Ag + m_bg + m_Bg - F_2$$

$$F_1 = g(m_A + m_b + m_B) - F_2$$

$$F_1 = 9.8m/s^2(10kg + 20kg + 10kg) - 343N$$

$$F_1 = 392N - 343N$$

$$F_1 = 49N$$

Last, balance the horizontal forces - not necessary in this problem.

Problem 2: A 15 kg mass hangs from the end of a 10 kg, 4 m uniform horizontal beam. The beam is attached to a wall by a cable and a hinge. See the figure. What is the tension in the cable, and the horizontal and vertical forces on the hinge?

Solution:

given:

Find the torques:

ARCHIMEDES' PRINCIPLE says that a upward buoyant force results from the pressure differential between the top and bottom of any object immersed in a fluid

Buoyancy explains why balloons float in the air and why ships float in water. If the maximum upward buoyant force is greater than or equal to the downward weight, an object will float. Displaced volume is the volume of the object surrounded by the fluid. If an object is not completely immersed, the object's total volume is not equal to the displaced volume. For example, ships have a draft (the height of the ship that is underwater) and a freeboard (the height of the ship that is above the water level).

If the density of an object is higher than the density of the fluid, the upward buoyant force is less than the downward weight, so the object will sink. However, it will feel lighter than normal.

Dynamic means moving – dynamic fluids are “flowing”, they are not at rest.

BERNOULLI'S PRINCIPLE relates the pressure and velocity changes of dynamic fluids, assuming LAMINAR FLOW - the fluid moves smoothly and is not TURBULENT.

Be aware, real fluid flow is usually turbulent, and what we do in this class is a very rough approximation.

Bernoulli's Principle is very useful, because it helps explain how an airplane flies. Assuming laminar (smooth) flow, if an airplane wing is cambered (curved) on the top

11 :

Problem 1: What is the weight of the air inside a living room of 4 m × 6 m × 3 m dimensions. The density of air is 1.20 kg/m³.

Solution: given: $V = 4m \times 6m \times 3m$, $\rho = 1.20kg/m^3$

unknown: $weight = mg = ?$

$$\rho = \frac{m}{V}$$

$$m = \rho V$$

$$mg = \rho g V$$

$$mg = 1.20kg/m^3 (9.8m/s^2) (4m \times 6m \times 3m)$$

$$mg = 847N \quad 190lb$$

Problem 2: Hoover Dam, on the Colorado River, controls Lake Mead, the largest reservoir in America (37 billion cubic meters of water), for irrigation and hydroelectric power for southern California, Nevada and Arizona. What would be the total force pushing water through a 2 m diameter underwater tunnel through the dam, 200 m below the water level. The density of fresh water is 1000kg/m³.

Solution: given: $r = \frac{d}{2} = 1m$, $h = 200m$, $\rho = 1000kg/m^3$

unknown: $F = ?$

Since $P = \frac{F}{A}$ and $P = \rho gh$

$$\frac{F}{A} = \rho gh$$

$$F = \rho gh A$$

then $F = \rho gh \pi r^2$

$$F = 1000kg/m^3 (9.8m/s^2) (200m) (3.14) (1m)^2$$

$$F = 6.15 \times 10^6 N$$

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Problem 3: A persistent complaint about the 1997 movie Titanic is near the end, when Jack saves Rose by pushing her on a door from the wreckage to use as a raft, but does not save himself by also getting on the door. Could Jack have climbed aboard the door

Problem 4: The largest airships ever built in the United States were USS's Akron and Macon of the 1930's US Navy. They were 785 ft long and held 6,500,000 cubic feet (184,000 cubic meters) of helium gas. Akron had a mass of 110,000 kg. What was her maximum useful load (crew, fuel, supplies)? The density of air and helium are 1.20 kg/m³ and 0.166 kg/m³, respectively.

Solution:

given:

$$m_{airship} = 110,000 \text{ kg}, V_{displaced} = V_{air} = V_{helium} = 184,000 \text{ m}^3,$$

$$\rho_{air} = 1.20 \text{ kg/m}^3, \rho_{helium} = 0.166 \text{ kg/m}^3$$

unknown: $m_{load} = ?$

$$F_{buoyant} = (mg)_{airship} + (mg)_{helium} + (mg)_{load}$$

(

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Problem 5: The largest twin-engine airplane ever made is the Boeing 777. It can carry 375 passengers over 8400 miles at 550 miles per hour (246 m/s). If half the total lift force on a 683,000 lb (310,000 kg) Triple-Seven cruising at 36,000 ft (11,000 m) altitude derives from Bernoulli's Principle, what would be the air velocity over the top of the wings with

12:

(1561 1626)

In this topic, "motion" still means moving, "harmonic" means the motion repeats back-and-forth, and "simple" means the motion only repeats, forever (theoretically). Simple

$$f = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$$

where k is the spring constant and m is the mass.

Since $f = \frac{1}{T}$, we can also write:

$$T = 2\pi \sqrt{\frac{m}{k}}$$

Conservation of energy gives us:

and

the definition of velocity gives us (PHY 1300 only):

and

And Newton's Second Law says

and

the definition of acceleration gives us (PHY 1300 only):

and

Note, all these formulae are stated without proof.

For a pendulum, with a small Amplitude ($A \ll L$), we have:

OR

where L is the pendulum cord length.

12

Problem 1: A 300. g stone is suspended from a vertical spring and set in motion. Measurements show the maximum speed of the vibrating stone is 35.0 cm/s and the period is 0.400 s. What are the (a) spring constant of the spring, (b) amplitude of the motion, and (c) frequency of oscillation.

Solution:

given: $m = 0.300\text{kg}$, $v_{max} = 0.350\text{m/s}$, $T = 0.400\text{s}$

unknown: $k = ?$, $A = ?$, $f = ?$

part (a):



part (b):

part (c):

Problem 2: A loudspeaker diaphragm is observed to be vibrating in simple harmonic motion with a total displacement of 0.50 mm while playing note middle C, 262 Hz. What are the (a) angular frequency, (b) maximum speed, and (c) magnitude of the maximum acceleration?

Solution:

given: $A = \frac{d}{2} = 0.25 \text{ mm} = 1.25 \times 10^{-4} \text{ m}$, $f = 262 \text{ Hz}$

unknown: $\omega = ?$, $v_{max} = ?$, $a_{max} = ?$

part (a):

$$\begin{aligned} &= 2\pi f \\ &= 2\pi (262 \text{ Hz}) \\ &1650 \text{ rad/s} \end{aligned}$$

part (b):

$$\begin{aligned} v_{max} &= 2\pi f A \\ v_{max} &= 2\pi (262 \text{ Hz}) (1.25 \times 10^{-4} \text{ m}) \\ v_{max} &0.206 \text{ m/s} \end{aligned}$$

part (c):

$$\begin{aligned} a_{max} &= 4\pi^2 f^2 A \\ a_{max} &= 4\pi^2 (262 \text{ Hz})^2 (1.25 \times 10^{-4} \text{ m}) \\ a_{max} &339 \text{ m/s}^2 \end{aligned}$$

13:

; . , □ (1835-1910)

There is a difference between how temperature is defined depending on whether we are talking about the macroscopic world ("large" things) versus the microscopic world ("small" things, usually atomic/molecular scale).

Macroscopic temperature measures heat stored in a collective object. Heat is a form of energy that is usually considered "inaccessible", because it has a natural "flow" (from hot to cold, by radiation, conduction or convection) that is difficult to interrupt and use to do work.

A large part of engineering is to develop machines to redirect the natural transfer of heat and do useful work - the term is "heat engine."

Microscopic temperature measures the kinetic energy atoms and molecules. On a microscopic scale, temperature is proportional to kinetic energy. We will not be worrying about this.

We are concerned with how heat affect the physical properties of matter.

CALORIMETRY:

A gain or loss of heat can change temperature, depending on a substance's specific heat - s .

$$Q = m s \Delta T$$

A gain or loss of heat can change phase (solid/liquid/gas), depending on a substance's heat of vaporization or fusion - H .

$$Q = m H$$

Since Chemistry is the study of different substances, calorimetry is also discussed there.

THERMAL EXPANSION:

A gain or loss of heat can change the length of an object, depending on a substance's linear expansion constant - α .

$$L = L_0 \alpha \Delta T$$

A gain or loss of heat can change the volume of an object depending on a substance's volume expansion constant - β .

$$V = V_0 \beta \Delta T$$

Problem 3: The coefficient of volume expansion of water is _____, and steel is _____. Since water's coefficient is higher, it expands faster than steel. If

$$\sin \theta = \frac{\text{opp}}{\text{hyp}} \quad \cos \theta = \frac{\text{adj}}{\text{hyp}} \quad \tan \theta = \frac{\text{opp}}{\text{adj}}$$

$$A_x = |A| \cos \theta_A \quad A_y = |A| \sin \theta_A$$

$$|A| = \sqrt{A_x^2 + A_y^2} \quad \theta_A = \arctan \left| \frac{A_y}{A_x} \right| \quad \tan^{-1} \left| \frac{A_y}{A_x} \right|$$

$$\vec{A} + \vec{B} = \vec{R}$$

$$A_x + B_x = R_x = |A| \cos \theta_A + |B| \cos \theta_B$$

$$A_y + B_y = R_y = |A| \sin \theta_A + |B| \sin \theta_B$$

$$\vec{A} \cdot \vec{B} = R = |A| |B| \cos \phi_{AB}$$

$$\vec{A} \cdot \vec{B} = R = A_x B_x + A_y B_y + A_z B_z$$

$$\vec{A} \times$$

$$T_{\text{satellite}} = \frac{2\pi r_{\text{orbit}}}{v_{\text{satellite}}}$$

$$W = F d = F d \cos \phi_{Fd}$$

$$KE = \frac{1}{2} m v^2$$

$$PE_{\text{grav}} = mgh$$

$$F_{\text{elastic}} = -kx$$

$$PE_{\text{elastic}} = \frac{1}{2} k x^2$$

$$W_e$$

$$I_{disk} = \frac{1}{2}mr^2$$

$$I_{solid\ sphere} = \frac{2}{5}mr^2$$

